Q1 :-

Solution :

The gradient of f at a point (x_0, y_0) where $f(x_0, y_0) = k$ is orthogonal to the curve $f(x, y) = k$ and points in the direction of increasing f.

Therefore :

- I - C
- II - B
- III - D
- IV - A

Q2 :-

Total work is zero on C_1, C_2.

First consider the path C_2, which is smooth. We claim that $\int_{C_2} F \cdot dr = 0$. On the first piece pointing downwards, work is negative since the constant force field F is antiparallel to the direction of movement of particle. Then on second piece work is zero since the particle is moving in a direction which is perpendicular to the force. Finally, on the third piece, work is positive. Since the first and third piece are anti-parallel and of same length and since vector field is constant. then the work done on the first and third pieces are cancelling out each other.

The path C_1 is the upper half of a circle. Splitting C_1 into the half left and half right, we can use similar arguments as above to see that the work done on the left half of C_1 cancels out the work done on the right half of C_1, resulting in a total work of zero along C_1.

Q4:-

Solution:-

area $A = \int_a^b y\,dx$

suppose $x = f(t)$ and $y = g(t)$, $t \in [0, 2\pi]$

Then Area $A = \int_a^b y\,dx = \int_0^{2\pi} g(t) f'(t)\,dt$

$$= \int_0^{2\pi} \cos t \cdot (t\cos t + \sin t \cdot 1)\,dt$$

$$= \int_0^{2\pi} t\cos^2 t\,dt + \int_0^{2\pi} \sin t \cos t\,dt$$

$$= \int_0^{2\pi} t\left(\frac{\cos 2t + 1}{2}\right) dt + \frac{1}{2}\int_0^{2\pi} \sin 2t\,dt$$

$$= \frac{1}{2}\left[\int_0^{2\pi} t\cos 2t\,dt + \int_0^{2\pi} t\,dt\right] + \frac{1}{2}\left(\frac{-\cos 2t}{2}\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}\left[\frac{t\sin 2t}{2}\Big|_0^{2\pi} - \int_0^{2\pi} 1 \cdot \frac{\sin 2t}{2}\,dt\right] +$$

$$\frac{1}{4} \times 4\pi^2 - \frac{1}{4}\left[\cos 4\pi - \cos 0\right]$$

$$= 0 + \pi^2 - 0$$

$= \pi^2$ unit, is the required area of domain.

Q5 :-

$$\nabla \times (\nabla \times \bar{F}) = \sum \bar{i} \times \frac{d}{dx} (\nabla \times \bar{F})$$

Now $\bar{i} \times \frac{d}{dx}(\nabla \times \bar{F}) = \bar{i} \times d/dx \left(\bar{i} \times \frac{d\bar{F}}{dx} + \bar{J} \times \frac{d\bar{F}}{dy} + \bar{K} \times \frac{d\bar{F}}{dz} \right)$

$$= i \times \left(i \times \frac{d^2 F}{dx^2} + \bar{J} \times \frac{d^2 \bar{F}}{dx\,dy} + \bar{K} \times \frac{d^2 F}{dx\,dz} \right)$$

$$= \bar{i} \times \left(\bar{i} \times \frac{d^2 \bar{F}}{dx^2} \right) + \bar{i} \times \left(\bar{J} \times \frac{d^2 \bar{F}}{dx\,dy} \right) + \bar{i} \times \left(\bar{K} \times \frac{d^2 F}{dx\,dz} \right)$$

$$= \left(\bar{i} \cdot \frac{d^2 F}{dx^2} \right) \bar{i} - \frac{d^2 F}{dx^2} + \left(\bar{i} \cdot \frac{d^2 F}{dx\,dy} \right) \bar{J} + \left(\bar{i} \cdot \frac{d^2 F}{dx\,dz} \right) \bar{K}$$

$$= \bar{i} \frac{d}{dx} \left(\bar{i} \cdot \frac{d\bar{F}}{dx} \right) + \bar{J} \frac{d}{dy} \left(\bar{i} \cdot \frac{d\bar{F}}{dx} \right) + \bar{K} \frac{d}{dz} \left(\bar{i} \cdot \frac{d\bar{F}}{dz} \right) - \frac{d^2 \bar{F}}{dx^2}$$

$$= \nabla \left(i \cdot \frac{d\bar{F}}{dx} \right) - \frac{d^2 \bar{F}}{dx^2}$$

$$\therefore \sum i \times \frac{d}{dx} \left(\nabla \times \bar{F} \right) = \nabla \sum i \cdot \frac{dF}{dx} - \sum \frac{d^2 F}{dx^2}$$

$$= \nabla (\nabla \cdot F) - \nabla F$$

Q3 :-

Solution (a) :-

(a)

$$\vec{F}\cdot d\vec{r} = 2xye^{y}dx + (x^2(y+1)e^{y} + 2y)dy$$

along line from (0,0) to (1,0) , $y = dy = 0$, $\vec{F}\cdot d\vec{r} = 0$

From (1,0) to (0,1) , $y = 1-x$, $dy = -dx$

$$\rightarrow \vec{F}\cdot d\vec{r} = 2x(1-x)e^{1-x}dx + (x^2(2-x)e^{1-x} + 2 - 2x)(-dx)$$

$$= \left[(2x-2x^2)e^{1-x} - (2x^2 - x^3)e^{1-x} - 2 + 2x\right]dx$$

$$= \left[(x^3 - 4x^2 + 2x)e^{1-x} - 2 + 2x\right]dx$$

$$\rightarrow \int_C \vec{F}\cdot d\vec{r} = \int_1^0 \left[(x^2 - 4x^2 + 2x)e^{1-x} - 2 + 2x\right]dx$$

$$= \Big[-(x^3 - 4x^2 + 2x)e^{1-x} - (3x^2 - 8x + 2)e^{1-x} \Big]^0$$

3 (b)

Along the straight line $y = 2x$

$\therefore dy = 2dx$

$\vec{F} \cdot d\vec{r} = (x+y)dx + 2y\,dy$

$= 3x\,dx + 8x\,dx$

$= 11x\,dx$

$\Rightarrow \int_c \vec{F} \cdot d\vec{r} = \int_0^1 11x\,dx$

$= 11 \left.\frac{x^2}{2}\right|_0^1$

$= \boxed{11/2}$

7/9

3 (c)

$$\int_C \vec{F}\cdot dr = \int_{OA} \vec{F}\cdot d\vec{r} + \int_{AB} \cdot\vec{F}\cdot d\vec{r} + \int_{BO} \vec{F}\cdot d\vec{r}$$

$$\vec{F} = xe^{2y}\hat{i} + x^2e^{2y}\hat{j}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$
$$d\vec{r} = dx\,i + dy\,j$$

$$\vec{F}\cdot dr = (xe^{2y}i + x^2e^{2y}j)(dx\,i + dy\,j)$$
$$\vec{F}\cdot dr = xe^{2y} + x^2e^{2y}dy$$

$$\int_{OA} \cdot\vec{F}\cdot d\vec{r} = \int_{OA} xe^{2y}dx + x^2e^{2y}dy$$

for OA $= x = 0 \quad dx = 0 \quad y = 2$

$$\int_{OA} \vec{F}\cdot d\vec{r} = \int_{OA} 0 \times e^{2\times 2} \times 0 + 0 \times e^{2\times 2} \times dy = 0$$

for AB. $y = 2 \to 0$ $\quad x$ varies $0 \to 2$

$$x^2 + y^2 = 4$$
$$x^2 = 4 - y^2$$
$$2x\,dx = 0 - 2y\,dy$$

puting x term in y

$$x\,dx = -y\,dy$$

$$\int_{AB} \vec{F} \cdot d\vec{r} = \int_{AB} -ey^{2y}\,dy + (4-y^2)\,e^{2y}\,dy$$

$$= e^{2y} \int_{AB} (4 - y - y^2)\,dy$$

ILATE part b continue. 9/9

$$\int_{AD} \underbrace{(4-y-y^2)}_{I}\ \underbrace{e^{2y}}_{II} dy \qquad I\int II\,dy - \int\left[\frac{d(I)}{dy}\cdot\int II\,dy\right]dy$$

$$(4-y-y^2)\int_2^0 e^{2y}dy - \int_2^0\left[\frac{d(4-y-y^2)}{dy}\int_2^0 e^{2y}dy\right]dy$$

$$\left[(1-y-y^2)\times\frac{e^{2y}}{2}\right]_2^0 - \int_2^0 \underbrace{(-1-2y)}_{I}\ \underbrace{\frac{e^{2y}}{2}}_{II}dy$$

$$\left[(1-y-y^2)\times\frac{e^{2y}}{2}\right]_2^0 + (1+2y)\int_2^0\frac{e^{2y}}{2} - \int_2^0\left[\frac{d(1+2y)}{dy}\int_2^0\frac{e^{2y}}{2}\right]dy$$

$$\left[(1-y-y^2)\frac{e^{2y}}{2}\right]_2^0 + \left[(1+2y)\frac{e^{2y}}{4}\right]_2^0 - \int_2^0 \cancel{2}\times\frac{e^{2y}}{\cancel{4}\,2}dy$$

$$\left[(1-y-y^2)\frac{e^{2y}}{2}\right]_2^0 + \left[(1+2y)\frac{e^{2y}}{4}\right]_2^0 - \left[\frac{e^{2y}}{4}\right]_2^0$$

$$\left[(1-0-0)\frac{e^0}{2} - (\cancel{1}-\cancel{2}-\cancel{4})\frac{e^4}{2}\right] + \left[(1+0)\frac{e^0}{4} - (1+2\times2)\frac{e^{2\times2}}{4}\right] - \left[\frac{e^{2\times0}}{4} - \frac{e^{2\times2}}{4}\right]$$

$$\frac{1}{2} + \frac{\cancel{5}e^4}{\cancel{2}} + \frac{\cancel{1}}{\cancel{4}} - \frac{5e^4}{4} - \frac{\cancel{1}}{\cancel{4}} + \frac{e^4}{4}$$

$2 + e^4 - e^4 = 2$

10/10

$\int_{BOC} \vec{F} \cdot d\vec{r} = \int_{BOC} x e^{2y} dx + x^2 e^{2y} dy$

x varies $2 \to 0$ and $y = 0$

$dy = 0$

$= \int_{BOC} x \times e^{2\times 0} dx + 0$

$= \int_2^0 x\,dx \Rightarrow \left[\frac{x^2}{2}\right]_2^0 = \frac{0}{2} - \frac{(2)^2}{2} = -2$

$\oint_C \vec{F} \cdot d\vec{r} = \int_{OA} \vec{F} \cdot d\vec{r} + \int_{AB} \vec{F} \cdot d\vec{r} + \int_{BO} \vec{F} \cdot d\vec{r}$

$0 + 2 - 2 = 0$